4. The Region Connection Calculus

The Region Connection Calculus (RCC) by Randell, Cui, and Cohn [138] and in particular RCC-8 is probably the best known approach to qualitative spatial reasoning. This is documented by the citations of RCC which appear in almost any paper on qualitative spatial reasoning and by the numerous research papers written on RCC itself. In this chapter we give a detailed introduction to the Region Connection Calculus and especially to RCC-8 which is the central topic of this book. A more detailed overview of the work on the Region Connection Calculus can be found in [28].

In the following section we summarize the original definitions and axioms of the Region Connection Calculus in first-order logic as given by Randell et al. [138]. Hence, the title of this section is the same as of Randell et al.'s paper. In Section 4.2 we introduce RCC-8 and give an encoding of RCC-8 in modal logic in Section 4.3 which is a slight modification of Bennett's original encoding [12]. In Section 4.4 we introduce Egenhofer's system of topological relations and compare it to the Region Connection Calculus.

4.1 A Spatial Logic Based on Regions and Connection

The Region Connection Calculus (RCC) developed by Randell, Cui, and Cohn [138] is a topological approach to spatial representation and reasoning where *spatial regions* are non-empty regular subsets of some topological space $\mathcal{U}$. Spatial regions do not have to be *internally connected*, i.e., they might consist of (multiple) disconnected pieces. Since all spatial regions are regular subsets of the same topological space $\mathcal{U}$, all spatial regions have the same dimension, namely, the dimension of $\mathcal{U}$ (provided that $\mathcal{U}$ has a particular dimension).

RCC is based on a single primitive relation between spatial regions, the "connected" relation C. The intended topological interpretation of $\mathsf{C}(a, b)$, where a and b are spatial regions, is that a and b are connected if and only if their topological closures share a common point. Within this interpretation it is not distinguished between open, semi-open, and closed regions which is different from previous approaches by Randell and Cohn [135, 134] and Clarke [23, 22]. The only requirements of the relation C is that it is reflexive and symmetric which is enforced by the following two axioms:

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$$\forall x \mathsf{C}(x, x) \tag{4.1}$$

$$\forall x, y[\mathsf{C}(x, y) \rightarrow \mathsf{C}(y, x)] \tag{4.2}$$

Using $\mathsf{C}(x, y)$, a large number of different relations can be defined. Among those are the following relations, their meaning under the intended interpretation of the C relation is given in brackets [138]: $\mathsf{P}(x, y)$ (x is a part of y), $\mathsf{PP}(x, y)$ (x is a proper part of y), $\mathsf{EQ}(x, y)$ (x is equal to y), $\mathsf{O}(x, y)$ (x overlaps y), $\mathsf{PO}(x, y)$ (x partially overlaps y), $\mathsf{DR}(x, y)$ (x is discrete from y), $\mathsf{EC}(x, y)$ (x is externally connected with y), $\mathsf{TPP}(x, y)$ (x is a tangential proper part of y), $\mathsf{NTPP}(x, y)$ (x is a non-tangential proper part of y). The relations $\mathsf{P}, \mathsf{PP}, \mathsf{TPP}$, and NTPP are non-symmetrical, their converses are denoted by $\mathsf{P}^{-1}, \mathsf{PP}^{-1}, \mathsf{TPP}^{-1}$, and NTPP^{-1}, respectively. The formal definition of these relations is the following [138]:

$$\mathsf{DC}(x, y) \equiv_{def} \neg\mathsf{C}(x, y) \tag{4.3}$$

$$\mathsf{P}(x, y) \equiv_{def} \forall z[\mathsf{C}(z, x) \rightarrow \mathsf{C}(z, y)] \tag{4.4}$$

$$\mathsf{PP}(x, y) \equiv_{def} \mathsf{P}(x, y) \wedge \neg\mathsf{P}(y, x) \tag{4.5}$$

$$\mathsf{EQ}(x, y) \equiv_{def} \mathsf{P}(x, y) \wedge \mathsf{P}(y, x) \tag{4.6}$$

$$\mathsf{O}(x, y) \equiv_{def} \exists z[\mathsf{P}(z, x) \wedge \mathsf{P}(z, y)] \tag{4.7}$$

$$\mathsf{PO}(x, y) \equiv_{def} \mathsf{O}(x, y) \wedge \neg\mathsf{P}(x, y) \wedge \neg\mathsf{P}(y, x) \tag{4.8}$$

$$\mathsf{DR}(x, y) \equiv_{def} \neg\mathsf{O}(x, y) \tag{4.9}$$

$$\mathsf{EC}(x, y) \equiv_{def} \mathsf{C}(x, y) \wedge \neg\mathsf{O}(x, y) \tag{4.10}$$

$$\mathsf{TPP}(x, y) \equiv_{def} \mathsf{PP}(x, y) \wedge \exists z[\mathsf{EC}(z, x) \wedge \mathsf{EC}(z, y)] \tag{4.11}$$

$$\mathsf{NTPP}(x, y) \equiv_{def} \mathsf{PP}(x, y) \wedge \neg\exists z[\mathsf{EC}(z, x) \wedge \mathsf{EC}(z, y)] \tag{4.12}$$

$$\mathsf{P}^{-1}(x, y) \equiv_{def} \mathsf{P}(y, x) \tag{4.13}$$

$$\mathsf{PP}^{-1}(x, y) \equiv_{def} \mathsf{PP}(y, x) \tag{4.14}$$

$$\mathsf{TPP}^{-1}(x, y) \equiv_{def} \mathsf{TPP}(y, x) \tag{4.15}$$

$$\mathsf{NTPP}^{-1}(x, y) \equiv_{def} \mathsf{NTPP}(y, x) \tag{4.16}$$

Using these relations, it is also possible to define Boolean functions such as $\mathsf{sum}(x,y)$ (the union of x and y), $\mathsf{compl}(x)$ (the complement of x), $\mathsf{prod}(x,y)$ (the intersection of x and y), and $\mathsf{diff}(x,y)$ (the difference of x and y). The functions compl, prod, and diff are partial since their result might be the empty region which is undefined. These functions can be used to define other relations such as internal connectedness of regions:

$$\mathsf{CON}(x) \equiv_{def} \forall y \forall z[\mathsf{EQ}(\mathsf{sum}(y, z), x) \rightarrow \mathsf{C}(y, z)] \tag{4.17}$$

Additional axioms can be used to specify properties of spatial regions. The following axiom states that every region has a non-tangential proper part, i.e., there are no atomic regions:

$$\forall x \exists y \mathsf{NTPP}(y, x) \tag{4.18}$$

Gotts [72] showed that every regular connected T_3 space is a model for the RCC axioms, i.e., every regular subset of such a topological space fulfills the axioms.

The relations, functions, and axioms presented here are just a small fraction of what can be expressed within the RCC theory. For instance, it is possible to add other primitive relations and functions such as the convex-hull function [138, 35]. Gotts [71, 73] studied a large number of different relations which can be defined upon the C relation. Of particular interest are those relations that form a set of jointly exhaustive and pairwise disjoint base relations. If these relations are closed under composition they generate a relation algebra, thus, reasoning about these relations can be done using the methods described in Section 2.4. What is needed is mainly a composition table which can be computed using the first-order definitions of the RCC relations. Depending on the level of granularity, many different sets of base relations can be defined within the RCC theory.

Randell et al. [138] suggested a set of eight base relations, later denoted as RCC-8. This set of relations is interesting for a number of reasons. It is the smallest set of base relations which allows topological distinctions rather than just mereological (being expressible by using the part-whole relationship). Most other relations definable in the RCC theory are refinements of these relations. As such, RCC-8 is ideally suited as a starting point for qualitative spatial reasoning which can be extended in many different ways. Furthermore, the semantics of these relations can be described by using propositional logics rather than first-order logics [10, 12], a property which allows us to prove decidability.

4.2 The Region Connection Calculus RCC-8

The Region Connection Calculus RCC-8 is the constraint language formed by the eight jointly exhaustive and pairwise disjoint base relations DC, EC, PO, EQ, TPP, NTPP, TPP^{-1}, and NTPP^{-1} definable in the RCC-theory and by all possible unions of the base relations. It can be easily verified by the first-order definitions given in the previous section that exactly one of the eight base relations holds between any two spatial regions. Unions of possible base relations are used to represent indefinite knowledge. Since the base relations are pairwise disjoint, this results in $2^8 = 256$ different RCC-8 relations altogether (including the empty relation and the universal relation). In some papers the set of base relations is denoted as RCC-8 while the set of all possible unions of base relations is denoted as 2^{RCC8}. We will, however, use RCC-8 to refer to the set of all possible disjunctions of the base relations and $\mathcal{B}$ to refer to the set of base relations. Analogous to the general RCC-theory, spatial regions in RCC-8 are non-empty regular subsets of some topological space that do not have to be internally connected, and do not have a particular dimension. Without loss of generality (due to the intended interpretation

Table 4.1. Topological interpretation of the eight base relations of RCC-8. All spatial regions are regular closed, i.e., $x = c(i(x))$ and $y = c(i(y))$. $i(\cdot)$ specifies the topological interior of a spatial region, $c(\cdot)$ the topological closure

RCC-8 *Relation*	*Topological Constraints*
$\mathsf{DC}(x,y)$	$x \cap y = \emptyset$
$\mathsf{EC}(x,y)$	$i(x) \cap i(y) = \emptyset,\ x \cap y \neq \emptyset$
$\mathsf{PO}(x,y)$	$i(x) \cap i(y) \neq \emptyset,\ x \not\subseteq y,\ y \not\subseteq x$
$\mathsf{TPP}(x,y)$	$x \subset y,\ x \not\subseteq i(y)$
$\mathsf{TPP}^{-1}(x,y)$	$y \subset x,\ y \not\subseteq i(x)$
$\mathsf{NTPP}(x,y)$	$x \subset i(y)$
$\mathsf{NTPP}^{-1}(x,y)$	$y \subset i(x)$
$\mathsf{EQ}(x,y)$	$x = y$

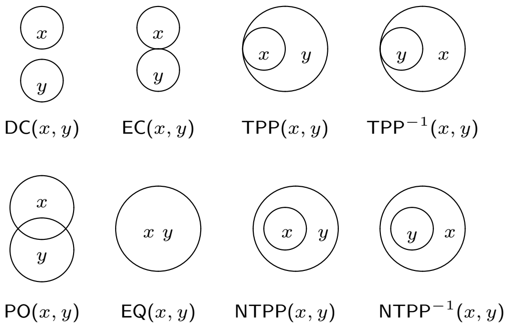

Fig. 4.1. Two-dimensional examples for the eight base relations of RCC-8

of the C relation) we require spatial regions to be regular *closed* subsets of a topological space.

The RCC-8 relations can be given a straightforward topological interpretation in terms of point-set topology (see Table 4.1), which is almost the same as for the topological relations given by Egenhofer [44] (though Egenhofer places stronger constraints on the domain of regions, e.g., regions must be one-piece and are not allowed to have holes, see Section 4.4). Examples for the RCC-8 base relations are given in Figure 4.1.

Converse, intersection and union of relations can easily be obtained by performing the corresponding set theoretic operations. Composition of base relations can be computed using the formal definitions of the relations given in the previous section [137, 10]. The compositions of the eight base relations are shown in Table 4.2. Every entry in the composition table specifies the relation obtained by composing the base relation of the corresponding row with the base relation of the corresponding column. Composition of two disjunctive RCC-8 relations can be obtained by computing the union of the composition of the base relations. Note that the composition table corresponds to the ex-

Table 4.2. Composition table for the eight base relations of RCC-8, where $*$ specifies the universal relation

$\circ$	DC	EC	PO	TPP	NTPP	TPP^{-1}	NTPP^{-1}	EQ
DC	$*$	DC,EC,PO,TPP,NTPP	DC,EC,PO,TPP,NTPP	DC,EC,PO,TPP,NTPP	DC,EC,PO,TPP,NTPP	DC	DC	DC
EC	DC,EC,PO,TPP^{-1},NTPP^{-1}	DC,EC,PO,TPP,TPP^{-1},EQ	DC,EC,PO,TPP,NTPP	EC,PO,TPP,NTPP	PO,TPP,NTPP	DC,EC	DC	EC
PO	DC,EC,PO,TPP^{-1},NTPP^{-1}	DC,EC,PO,TPP^{-1},NTPP^{-1}	$*$	PO,TPP,NTPP	PO,TPP,NTPP	DC,EC,PO,TPP^{-1},NTPP^{-1}	DC,EC,PO,TPP^{-1},NTPP^{-1}	PO
TPP	DC	DC,EC	DC,EC,PO,TPP,NTPP	TPP,NTPP	NTPP	DC,EC,PO,TPP,TPP^{-1},EQ	DC,EC,PO,TPP^{-1},NTPP^{-1}	TPP
NTPP	DC	DC	DC,EC,PO,TPP,NTPP	NTPP	NTPP	DC,EC,PO,TPP,NTPP	$*$	NTPP
TPP^{-1}	DC,EC,PO,TPP^{-1},NTPP^{-1}	EC,PO,TPP^{-1},NTPP^{-1}	PO,TPP^{-1},NTPP^{-1}	PO,EQ,TPP,TPP^{-1}	PO,TPP,NTPP	TPP^{-1},NTPP^{-1}	NTPP^{-1}	TPP^{-1}
NTPP^{-1}	DC,EC,PO,TPP^{-1},NTPP^{-1}	PO,TPP^{-1},NTPP^{-1}	PO,TPP^{-1},NTPP^{-1}	PO,TPP^{-1},NTPP^{-1}	PO,TPP^{-1},TPP,NTPP,NTPP^{-1},EQ	NTPP^{-1}	NTPP^{-1}	NTPP^{-1}
EQ	DC	EC	PO	TPP	NTPP	TPP^{-1}	NTPP^{-1}	EQ

tensional definition of composition given in Section 2.4.1 only if the universal region is not permitted [15].

A spatial configuration can be described by specifying a set Θ of constraints over RCC-8, written as xRy or $R(x, y)$, where R is an RCC-8 relation and x, y are *spatial variables* over the infinite domain of all possible spatial regions. An important reasoning problem is deciding consistency of Θ, i.e., deciding whether there is an assignment of non-empty, regular closed regions of some topological space to variables of Θ in a way that all constraints are satisfied. We call this problem RSAT, or RSAT($\mathcal{S}$) if only relations of a specific set $\mathcal{S}$ are used in Θ. RSAT is a sub-problem of CSPSAT which is defined in Section 2.4.2 and which can be tackled using the same methods, for instance, enforcing path-consistency as a partial method for deciding consistency. Other useful reasoning problems include the minimal labels problem RMIN and the entailment problem RENT, the sub-problems of CSPMIN and CSPENT, respectively.

Another set of jointly exhaustive and pairwise disjoint base relations definable in the RCC-theory on a coarser level of granularity than RCC-8 is RCC-5 [10]. For RCC-5 the boundary of a region is not taken into account, i.e., one does not distinguish between DC and EC and between TPP and NTPP. These relations are combined to the RCC-5 base relations DR and PP, respectively (see Section 4.1). Thus, RCC-5 contains the five base relations DR, PO, PP, PP^{-1}, and EQ and all 2^5 possible disjunctions thereof. The RCC-5 relations are closed under composition and form a relation algebra. The composition table for RCC-5 is given in Table 4.3. In this work we will focus on RCC-8,

Table 4.3. Composition table for the five base relations of RCC-5

$\circ$	DR	PO	PP	PP^{-1}	EQ
DR	$*$	DR,PO,PP	DR,PO,PP	DR	DR
PO	DR,PO,PP^{-1}	$*$	PO,PP	DR,PO,PP^{-1}	PO
PP	DR	DR,PO,PP	PP	$*$	PP
PP^{-1}	DR,PO,PP^{-1}	PO,PP^{-1}	PO,PP,PP^{-1},EQ	PP^{-1}	PP^{-1}
EQ	DR	PO	PP	PP^{-1}	EQ

but most of our results can easily be applied to RCC-5.

4.3 Encoding of RCC-8 in Modal Logic

Another way of solving problems concerning RCC-8 is using the encoding of the relations in first order logic. Such an encoding does not lead to efficient decision procedures, however. In order to overcome this problem, Bennett [10, 12] used different encodings of RCC-8 in propositional intuitionistic and modal logic. In this work we will use a slight modification of Bennett's encoding of RCC-8 in propositional modal logic [12]. An introduction to modal logics is given in Section 2.2.2.

Remark 4.1. Throughout this work we will use the following convention for referring to spatial regions, spatial variables, and propositional atoms corresponding to spatial regions or spatial variables:

– Spatial variables are written as x, y, z.
– Spatial regions are written as $\mathtt{X}, \mathtt{Y}, \mathtt{Z}$.
– Propositional atoms corresponding to spatial regions or spatial variables are written as $\mathsf{X}, \mathsf{Y}, \mathsf{Z}$.

If the same letter is used in different fonts in the same context, it represents the same region. For instance, $\mathtt{X}$ is a possible instance of x, $\mathtt{Y}$ a possible instance of y, and X is the propositional atom corresponding to x or to $\mathtt{X}$.

Bennett obtained the modal encoding by analyzing the relationship of regions to the universe $\mathcal{U}$. For the modal encoding we are using, Bennett restricted his analysis to closed regions that are connected if they share a point and overlap if they share an interior point.[1] If, e.g, $\mathtt{X}$ and $\mathtt{Y}$ are disconnected,

[1] There is also a modal encoding based on open regions which is not as simple as the encoding based on closed regions [12].

Table 4.4. Bennett's encoding of the eight base relations in modal logic [12]

Relation	*Model Constraints*	*Entailment Constraints*
$\mathsf{DC}(x,y)$	$\neg(\mathsf{X} \wedge \mathsf{Y})$	$\neg\mathsf{X}, \neg\mathsf{Y}$
$\mathsf{EC}(x,y)$	$\neg(\mathbf{I}\mathsf{X} \wedge \mathbf{I}\mathsf{Y})$	$\neg(\mathsf{X} \wedge \mathsf{Y}), \neg\mathsf{X}, \neg\mathsf{Y}$
$\mathsf{PO}(x,y)$	—	$\neg(\mathbf{I}\mathsf{X} \wedge \mathbf{I}\mathsf{Y}), \mathsf{X} \to \mathsf{Y}, \mathsf{Y} \to \mathsf{X}, \neg\mathsf{X}, \neg\mathsf{Y}$
$\mathsf{TPP}(x,y)$	$\mathsf{X} \to \mathsf{Y}$	$\mathsf{X} \to \mathbf{I}\mathsf{Y}, \mathsf{Y} \to \mathsf{X}, \neg\mathsf{X}, \neg\mathsf{Y}$
$\mathsf{TPP}^{-1}(x,y)$	$\mathsf{Y} \to \mathsf{X}$	$\mathsf{Y} \to \mathbf{I}\mathsf{X}, \mathsf{X} \to \mathsf{Y}, \neg\mathsf{X}, \neg\mathsf{Y}$
$\mathsf{NTPP}(x,y)$	$\mathsf{X} \to \mathbf{I}\mathsf{Y}$	$\mathsf{Y} \to \mathsf{X}, \neg\mathsf{X}, \neg\mathsf{Y}$
$\mathsf{NTPP}^{-1}(x,y)$	$\mathsf{Y} \to \mathbf{I}\mathsf{X}$	$\mathsf{X} \to \mathsf{Y}, \neg\mathsf{X}, \neg\mathsf{Y}$
$\mathsf{EQ}(x,y)$	$\mathsf{X} \to \mathsf{Y}, \mathsf{Y} \to \mathsf{X}$	$\neg\mathsf{X}, \neg\mathsf{Y}$

the complement of the intersection of $\mathtt{X}$ and $\mathtt{Y}$ is equal to the universe. Further, both regions must not be empty, i.e., the complements of both $\mathtt{X}$ and $\mathtt{Y}$ are not equal to the universe. In the same way all topological constraints corresponding to the RCC-8 relations (see Table 4.1) can be written as constraints of the form $(m = \mathcal{U})$ and $(e \neq \mathcal{U})$, where m and e are set-theoretic expressions, denoted as *model constraints* and *entailment constraints*, respectively [10]. In the above example, $\overline{\mathtt{X} \cap \mathtt{Y}}$ is the model constraint and $\overline{\mathtt{X}}$ and $\overline{\mathtt{Y}}$ are the entailment constraints. Any model constraint must hold, whereas no entailment constraint must hold [10].

For some of the constraints it is necessary to refer to the interior of regions. For this purpose the topological interior operator i is used. This operator must satisfy the following constraints for arbitrary sets $\Phi, \Psi \subseteq \mathcal{U}$ [12]:

$$i(\Phi) \subseteq \Phi, \tag{4.19}$$
$$i(i(\Phi)) = i(\Phi), \tag{4.20}$$
$$i(\mathcal{U}) = \mathcal{U}, \tag{4.21}$$
$$i(\Phi \cap \Psi) = i(\Phi) \cap i(\Psi). \tag{4.22}$$

The model and entailment constraints can be encoded in modal logic, where regions correspond to propositional atoms, the interior operator i corresponds to a modal operator $\mathbf{I}$ (see Table 4.4), and the universe $\mathcal{U}$ corresponds to the set of all worlds W [12]. The constraints for i must also hold for the modal operator $\mathbf{I}$, which results in the following axiom schemata [12] for arbitrary modal formulas ϕ, ψ:

$$\mathbf{I}\phi \to \phi, \tag{4.23}$$
$$\mathbf{I}\mathbf{I}\phi \leftrightarrow \mathbf{I}\phi, \tag{4.24}$$
$$\mathbf{I}\top \leftrightarrow \top \text{ (for any tautology } \top), \tag{4.25}$$
$$\mathbf{I}(\phi \wedge \psi) \leftrightarrow \mathbf{I}\phi \wedge \mathbf{I}\psi. \tag{4.26}$$

Axiom schemata 4.23 and 4.24 correspond to the modal axioms **T** and **4** and axiom schemata 4.26 and 4.26 already hold for any modal logic K, so **I** is a modal S4-operator (see Section 2.2.2).

The four axiom schemata specified by Bennett are not sufficient to exclude non-closed regions as it was intended. In order to account for that, we add one formula for each atom X, which corresponds to the topological property of regular closed regions:

A regular closed region is the closure of an open region. $\neg\mathsf{X}$ specifies the complement of $\mathtt{X}$, and, thus, $\neg\mathbf{I}\neg\mathsf{X}$ the closure of $\mathtt{X}$.

$$\mathsf{X} \leftrightarrow \neg\mathbf{I}\neg\mathbf{I}\mathsf{X} \tag{4.27}$$

Note that the S4 encoding can be used to reason about any kind of open or closed regions. Both the non-emptiness constraint, i.e., the entailment constraint $\neg\mathsf{X}$, and the regularity constraint (4.27) are optional and can be regarded as properties of regions definable in the modal representation. They are needed to make the representation conform to the intended interpretation of the original RCC theory.

In order to combine the different model and entailment constraints, Bennett [12] uses another modal operator $\Box$. $\Box\phi$ is interpreted as $\phi = \mathcal{U}$ and $\neg\Box\phi$ as $\phi \neq \mathcal{U}$. Since m is a model constraint if $m = \mathcal{U}$ holds, any model constraint m can be written as $\Box m$ and any entailment constraint e as $\neg\Box e$. If $\Box\mathsf{X}$ is true in a world w of a model $\mathcal{M}$, written as $\mathcal{M}, w \Vdash \Box\mathsf{X}$, then X must be true in any world of $\mathcal{M}$. So $\Box$ is an $\mathsf{S5}$-operator with the constraint that all worlds are mutually accessible. Therefore Bennett calls it a *strong* $\mathsf{S5}$-*operator* [12]. Now all model and all entailment constraints containing the strong $\mathsf{S5}$-operator can be conjunctively combined to a single modal formula. So the modal encoding of RCC-$\mathsf{8}$ is made with an $\mathsf{S4}$-operator that corresponds to the topological interior operator and a strong $\mathsf{S5}$-operator that is used to obtain a single modal formula.

4.4 Egenhofer's Approach to Topological Spatial Relations

Independently of the Region Connection Calculus a different approach to topological spatial relationship was developed by Egenhofer in the area of geographical information systems (GIS). Egenhofer [44] classified the relationships between two spatial entities according to the nine possible intersections of their interior, exterior, and boundary, hence, called the *9-intersection-model*. Depending on the nature of the considered spatial entities, many different relationships can be expressed by this model. For instance, it is possible to use spatial entities of different dimensions or distinguish different degrees of intersection. In his first approach, however, Egenhofer restricted the domain of spatial entities to be two-dimensional spatial regions whose boundary is a closed Jordan curve, i.e., simply connected planar regions that are not allowed to have holes. When looking at only whether the nine intersections for this kind of spatial regions are empty or non-empty and when eliminating all impossible relations, this results in eight different binary topological

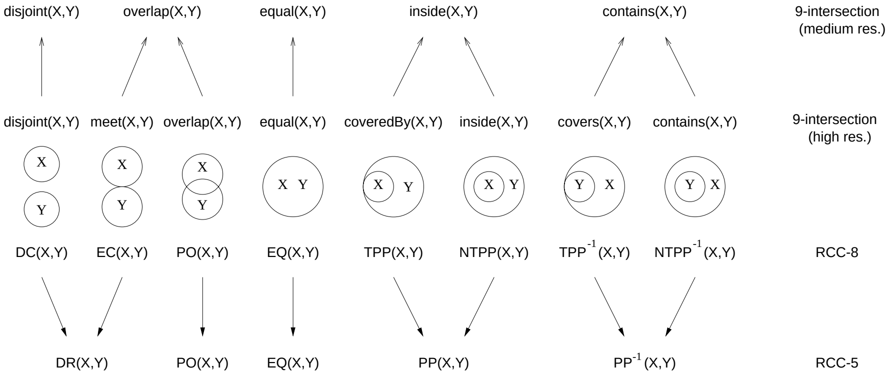

Fig. 4.2. Comparison of different systems of topological relations: the Region Connection Calculi RCC-8 and RCC-5 and the high and medium resolution of Egenhofer's 9-intersection model

relationships. The Region Connection Calculus and the 9-intersection model, which are two completely different approaches to topological relationships, lead to exactly (apart from the different constraints on regions) the same set of topological relations. Thus, there seems to be a natural agreement about what is a reasonable level of granularity of topological relations.

The computational properties of Egenhofer's topological relations were studied by Grigni et al. [77] who considered two different notions of satisfiability, the purely syntactical notion of *relational consistency* and the semantic notion of *realizability*, which are both different from what we call consistency. Relational consistency means that there is a path-consistent refinement of all relations to base relations, realizability means that there is a model consisting of simply connected planar regions. Orthogonal to this distinction, Grigni et al. [77] considered different sets of JEPD relations. One of them is the original set of eight base relations suggested by Egenhofer which they called the *high resolution* case. Another set consists of five base relations, the *medium resolution* case, which are obtained by combining some of the high resolution relations. Similar to RCC-5, the medium resolution relations do not distinguish relationships according to the boundary of regions. The difference of this set to RCC-5 is that the relations EC and PO are combined to form a new base relation, whereas for RCC-5 the relations DC and EC are combined (see Figure 4.2). With this distinction, the medium resolution relations can also be used to represent the possible relationships between non-topological sets: overlap(a, b) means that the sets a and b have a non-empty intersection while none of them is a subset of the other, if disjoint(a, b) holds, the sets a and b have no elements in common. This set-theoretic interpretation is not possible for RCC-5, since it distinguishes between interior and boundary elements of the sets. If two sets share some boundary elements, they are still in the DR relationship.

By reducing the NP-hard string-graph problem [105, 106], Grigni et al. [77] showed that deciding realizability is NP-hard for the high and medium resolution cases even if only constraints over the base relations are used. It is an open problem whether the realizability problem is in NP and even whether it is decidable. The reduction of the string graph problem, however, is possible only because all regions must be simply connected planar regions. Hence, this result does not carry over to the consistency problem of RCC-8 where spatial regions can be of any dimension and internal connectedness is not required. The relational consistency problem, which is obviously tractable if only base relations are used, was shown to be NP-hard for the high and medium resolution cases if all disjunctions over the base relations are permitted.