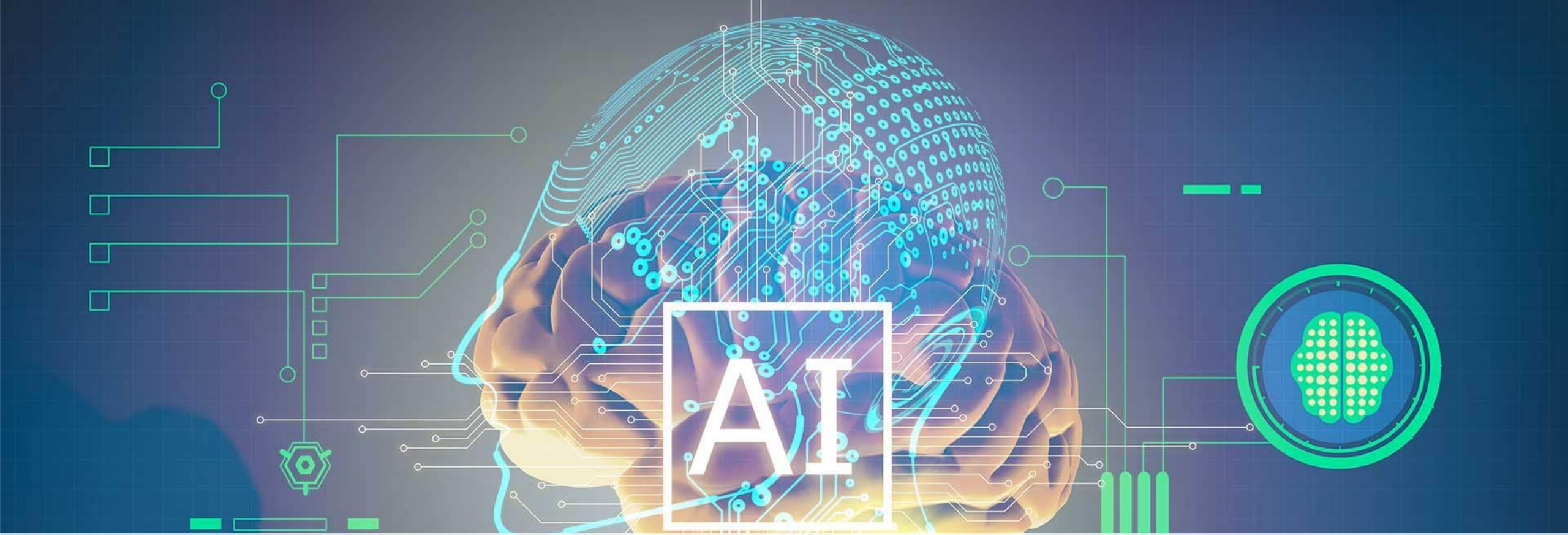
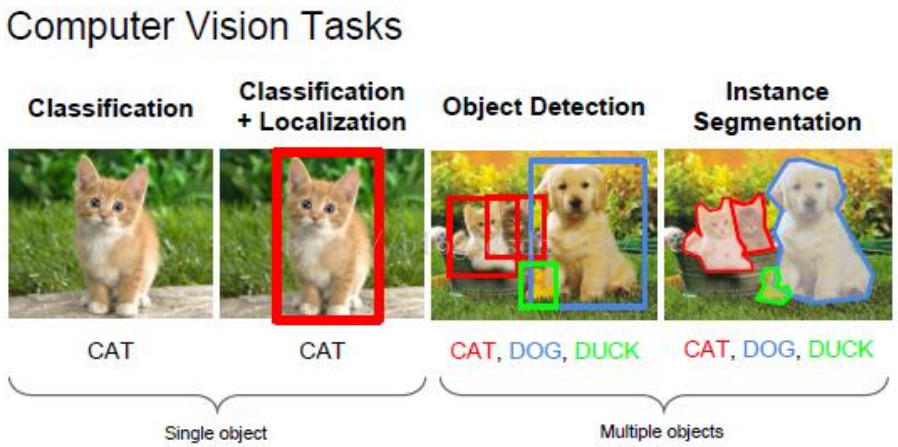


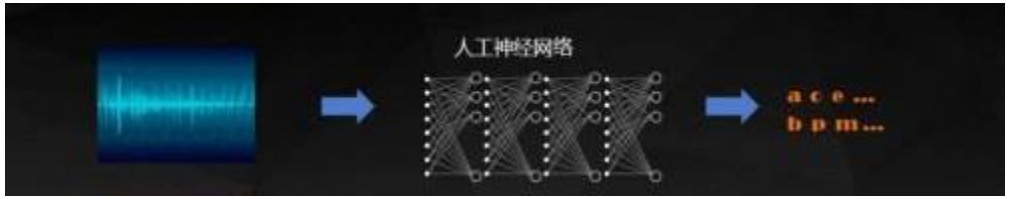
人工神经网络与深度学习

北京大学地空数算SESSDSA2019

深度学习应用场景



计算机视觉



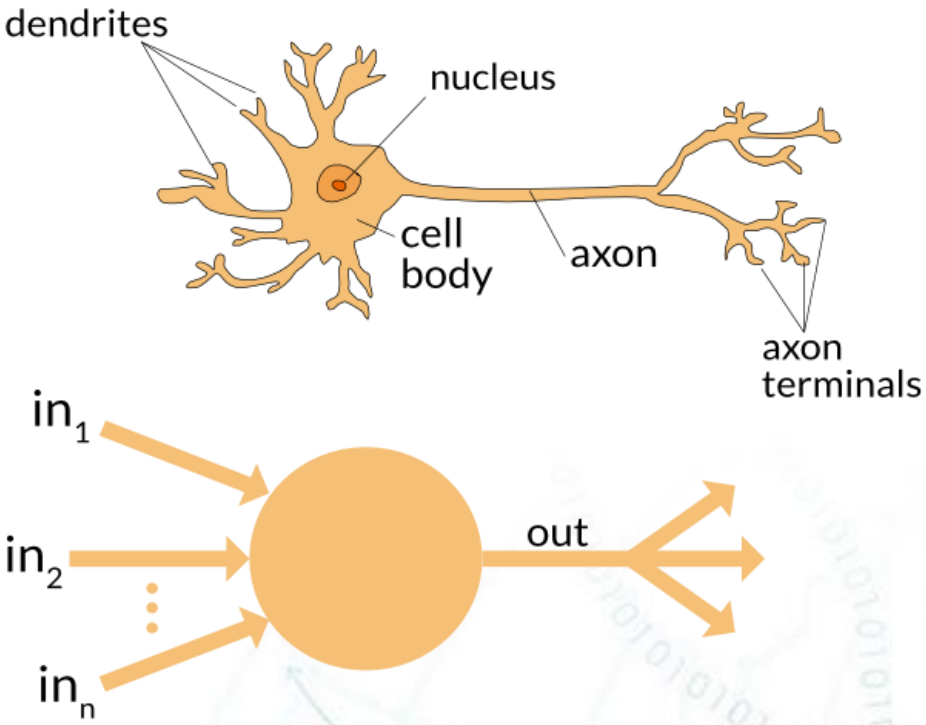
语音识别

自然语言处理

人机博弈

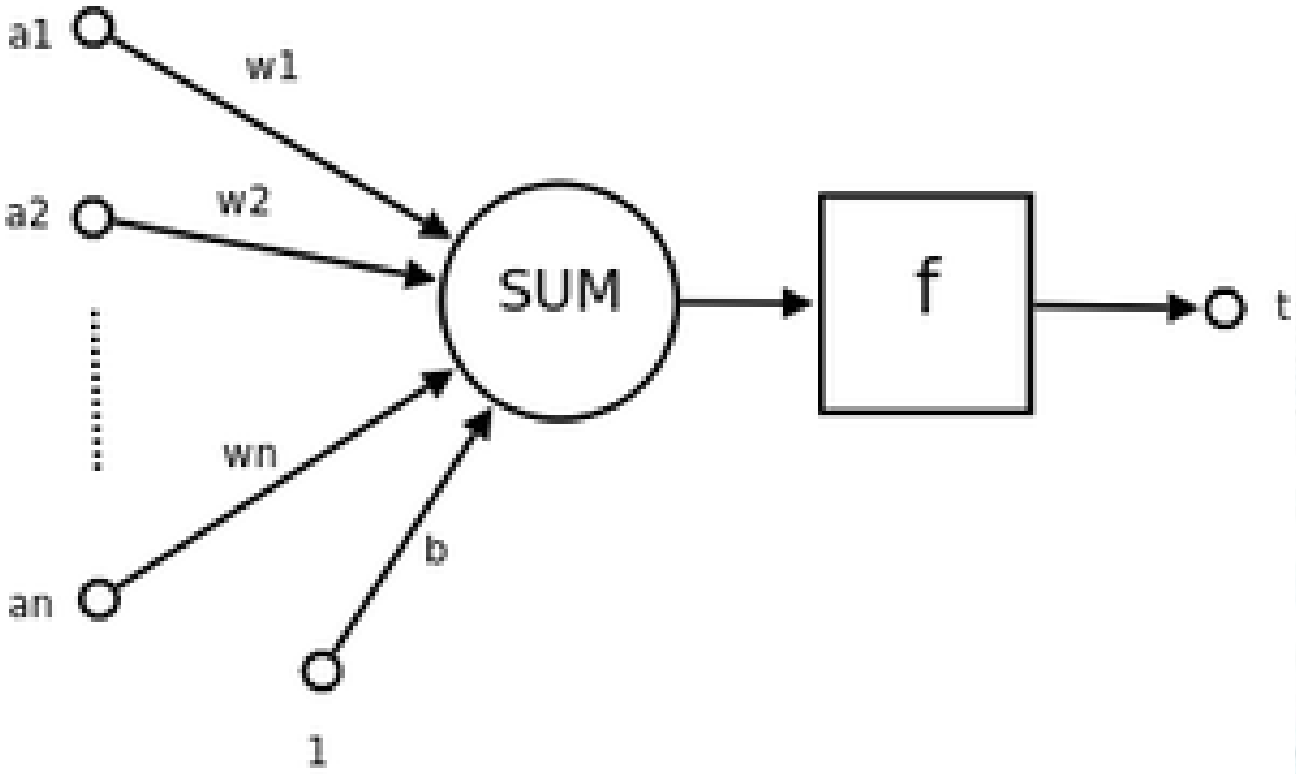
神经元

- 是生物神经细胞的简单抽象。
- 神经细胞结构大致可分为：树突、突触、细胞体及轴突。
- 单个神经细胞可被视为一种只有两种状态的机器——激动时为‘是’，而未激动时为‘否’。



神经元

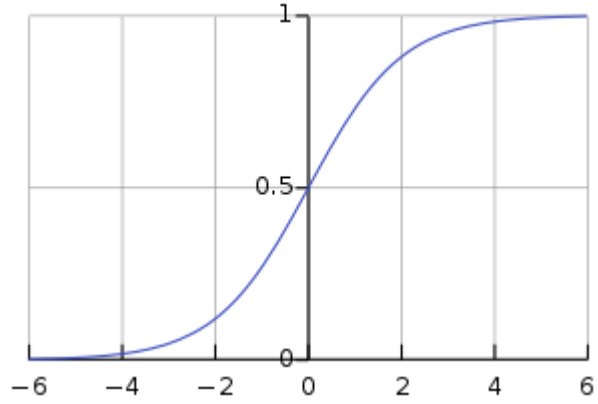
- 设$\vec{a} = (a_1, a_2, \ldots, a_n)$为这个输入的n维分量，$\vec{w} = (w_1, w_2, \ldots, w_n)$为权重，b为偏置量，$f(x)$为激活函数，即



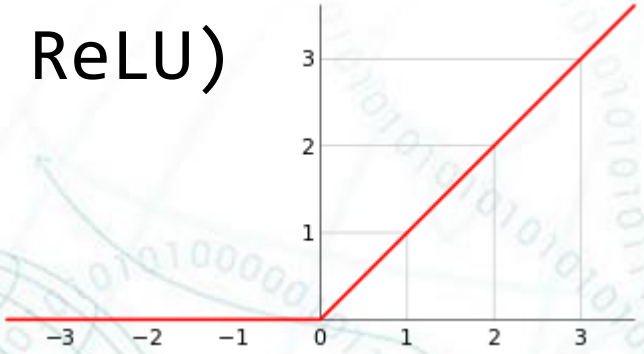
- $f(x) = \begin{cases} 1 \ if \ x > 0 \\ 0 \ if \ x \leq 0 \end{cases}$
- 那么预测值就是$\hat{y} = f(\vec{a} \cdot \vec{w} + b)$

激活函数

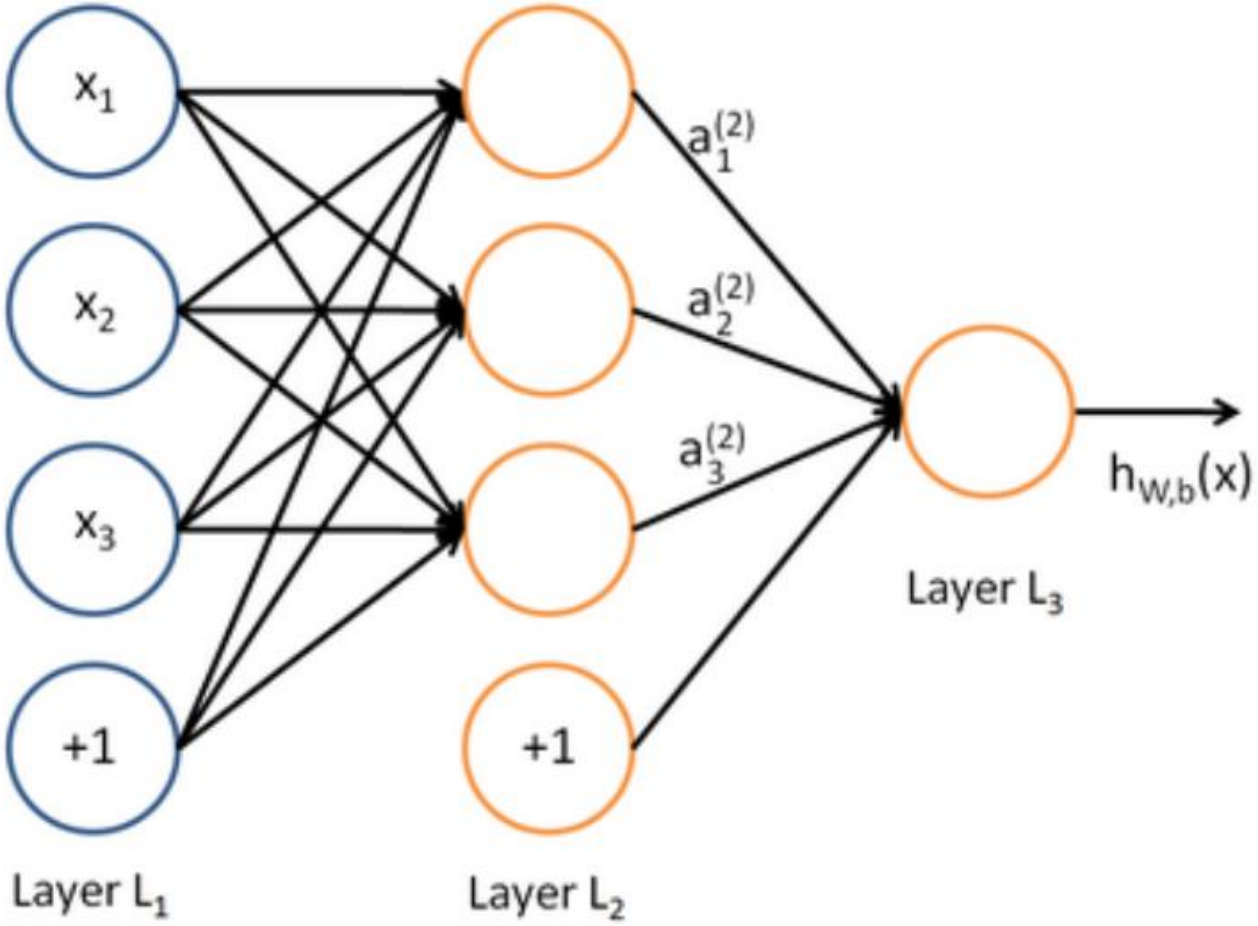
- Sigmoid function
- $\sigma(x) = \frac{1}{1+e^{-x}} = \frac{e^x}{e^x+1}$



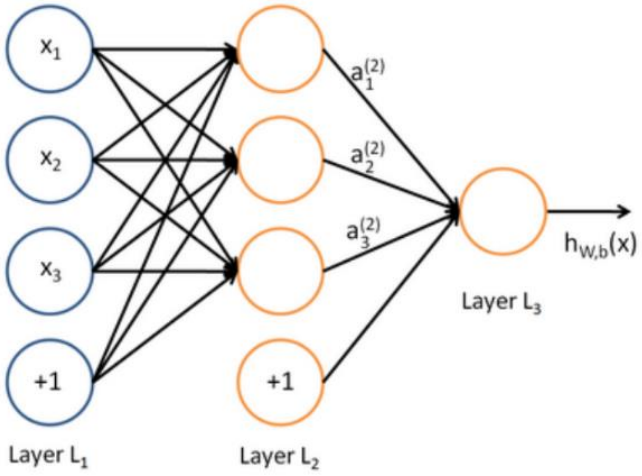
- 线性整流函数（Rectified Linear Unit, ReLU）
- $f(x) = \max(0, x)$



神经网络

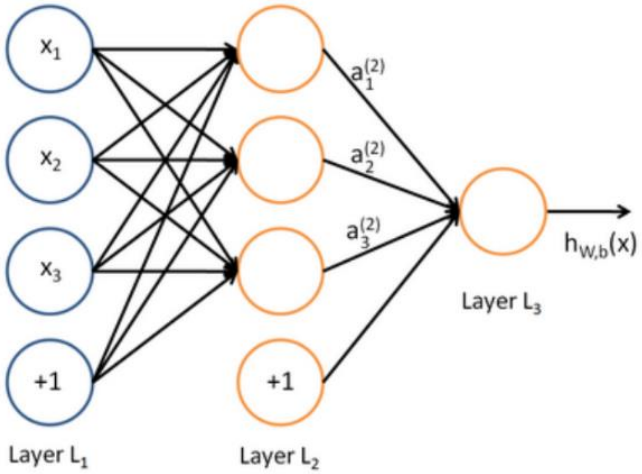


前向传播



$$
\begin{aligned}
a_1^{(2)} &= f(W_{11}^{(1)}x_1 + W_{12}^{(1)}x_2 + W_{13}^{(1)}x_3 + b_1^{(1)}) \\
a_2^{(2)} &= f(W_{21}^{(1)}x_1 + W_{22}^{(1)}x_2 + W_{23}^{(1)}x_3 + b_2^{(1)}) \\
a_3^{(2)} &= f(W_{31}^{(1)}x_1 + W_{32}^{(1)}x_2 + W_{33}^{(1)}x_3 + b_3^{(1)}) \\
h_{W,b}(x) = a_1^{(3)} &= f(W_{11}^{(2)}a_1^{(2)} + W_{12}^{(2)}a_2^{(2)} + W_{13}^{(2)}a_3^{(2)} + b_1^{(2)})
\end{aligned}
$$

前向传播



$$z^{(2)} = W^{(1)}x + b^{(1)}$$

$$a^{(2)} = f(z^{(2)})$$

$$z^{(3)} = W^{(2)}a^{(2)} + b^{(2)}$$

$$h_{W,b}(x) = a^{(3)} = f(z^{(3)})$$

损失函数

- 交叉熵（cross entropy）
  - $\mathrm{CE}(Y, f(x)) = -\sum_{x \in X} Y \log f(x)$
- 平方损失函数
  - $L(Y, f(x)) = \sum_{x \in X} (Y - f(x))^2$

反向传播

假设我们有一个固定样本集 $\{(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})\}$ ，它包含m个样例。我们可以用批量梯度下降法来求解神经网络。具体来讲，对于单个样例 (x, y) ，其代价函数为：

$$J(W, b; x, y) = \frac{1}{2} \left\| h_{W,b}(x) - y \right\|^2 .$$

反向传播

给定一个包含 m 个样例的数据集，我们可以定义整体代价函数为：

$$
\begin{aligned}
J(W,b) &= \left[\frac{1}{m}\sum_{i=1}^{m} J(W,b;x^{(i)},y^{(i)})\right] + \frac{\lambda}{2}\sum_{l=1}^{n_l-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}}\left(W_{ji}^{(l)}\right)^2 \\
&= \left[\frac{1}{m}\sum_{i=1}^{m}\left(\frac{1}{2}\left\|h_{W,b}(x^{(i)})-y^{(i)}\right\|^2\right)\right] + \frac{\lambda}{2}\sum_{l=1}^{n_l-1}\sum_{i=1}^{s_l}\sum_{j=1}^{s_{l+1}}\left(W_{ji}^{(l)}\right)^2
\end{aligned}
$$

反向传播

梯度下降法中每一次迭代都按照如下公式对参数W和b进行更新：

$$W_{ij}^{(l)} = W_{ij}^{(l)} - \alpha \frac{\partial}{\partial W_{ij}^{(l)}} J(W, b)$$

$$b_i^{(l)} = b_i^{(l)} - \alpha \frac{\partial}{\partial b_i^{(l)}} J(W, b)$$

反向传播

首先看如何使用反向传播算法来计算 $\frac{\partial}{\partial W_{ij}^{(l)}} J(W,b;x,y)$ 和 $\frac{\partial}{\partial b_i^{(l)}} J(W,b;x,y)$，这两项是单个样例 (x,y) 的代价函数 $J(W,b;x,y)$ 的偏导数。一旦我们求出该偏导数，就可以推导出整体代价函数 $J(W,b)$ 的偏导数：

$$\frac{\partial}{\partial W_{ij}^{(l)}} J(W,b) = \left[\frac{1}{m} \sum_{i=1}^{m} \frac{\partial}{\partial W_{ij}^{(l)}} J(W,b;x^{(i)},y^{(i)}) \right] + \lambda W_{ij}^{(l)}$$

$$\frac{\partial}{\partial b_i^{(l)}} J(W,b) = \frac{1}{m} \sum_{i=1}^{m} \frac{\partial}{\partial b_i^{(l)}} J(W,b;x^{(i)},y^{(i)})$$

反向传播

1.进行前馈传导计算，利用前向传导公式，得到 $L_2, L_3, \ldots$ 直到输出层 L_{n_l} 的激活值。

2.对于第 n_l 层（输出层）的每个输出单元 i ，我们根据以下公式计算残差：

$$\delta_i^{(n_l)} = \frac{\partial}{\partial z_i^{(n_l)}} \frac{1}{2} \left\| y - h_{W,b}(x) \right\|^2 = -(y_i - a_i^{(n_l)}) \cdot f'(z_i^{(n_l)})$$

$$\begin{aligned}
\delta_i^{(n_l)} &= \frac{\partial}{\partial z_i^{n_l}} J(W,b;x,y) = \frac{\partial}{\partial z_i^{n_l}} \frac{1}{2} \left\| y - h_{W,b}(x) \right\|^2 \\
&= \frac{\partial}{\partial z_i^{n_l}} \frac{1}{2} \sum_{j=1}^{S_{n_l}} (y_j - a_j^{(n_l)})^2 = \frac{\partial}{\partial z_i^{n_l}} \frac{1}{2} \sum_{j=1}^{S_{n_l}} (y_j - f(z_j^{(n_l)}))^2 \\
&= -(y_i - f(z_i^{(n_l)})) \cdot f'(z_i^{(n_l)}) = -(y_i - a_i^{(n_l)}) \cdot f'(z_i^{(n_l)})
\end{aligned}$$

反向传播

3. 对 $l = n_l - 1, n_l - 2, n_l - 3, \ldots, 2$ 的各个层，第 l 层的第 i 个节点的残差计算方法如下：

$$\delta_i^{(l)} = \left(\sum_{j=1}^{s_{l+1}} W_{ji}^{(l)} \delta_j^{(l+1)} \right) f'(z_i^{(l)})$$

反向传播

$$
\begin{aligned}
\delta_i^{(n_l-1)} &= \frac{\partial}{\partial z_i^{n_l-1}} J(W,b;x,y) = \frac{\partial}{\partial z_i^{n_l-1}} \frac{1}{2} \left\| y - h_{W,b}(x) \right\|^2 = \frac{\partial}{\partial z_i^{n_l-1}} \frac{1}{2} \sum_{j=1}^{S_{n_l}} (y_j - a_j^{(n_l)})^2 \\
&= \frac{1}{2} \sum_{j=1}^{S_{n_l}} \frac{\partial}{\partial z_i^{n_l-1}} (y_j - a_j^{(n_l)})^2 = \frac{1}{2} \sum_{j=1}^{S_{n_l}} \frac{\partial}{\partial z_i^{n_l-1}} (y_j - f(z_j^{(n_l)}))^2 \\
&= \sum_{j=1}^{S_{n_l}} -(y_j - f(z_j^{(n_l)})) \cdot \frac{\partial}{\partial z_i^{(n_l-1)}} f(z_j^{(n_l)}) = \sum_{j=1}^{S_{n_l}} -(y_j - f(z_j^{(n_l)})) \cdot f'(z_j^{(n_l)}) \cdot \frac{\partial z_j^{(n_l)}}{\partial z_i^{(n_l-1)}} \\
&= \sum_{j=1}^{S_{n_l}} \delta_j^{(n_l)} \cdot \frac{\partial z_j^{(n_l)}}{\partial z_i^{n_l-1}} = \sum_{j=1}^{S_{n_l}} \left(\delta_j^{(n_l)} \cdot \frac{\partial}{\partial z_i^{n_l-1}} \sum_{k=1}^{S_{n_l-1}} f(z_k^{n_l-1}) \cdot W_{jk}^{n_l-1} \right) \\
&= \sum_{j=1}^{S_{n_l}} \delta_j^{(n_l)} \cdot W_{ji}^{n_l-1} \cdot f'(z_i^{n_l-1}) = \left(\sum_{j=1}^{S_{n_l}} W_{ji}^{n_l-1} \delta_j^{(n_l)} \right) f'(z_i^{n_l-1})
\end{aligned}
$$

将上式中的$n_l - 1$与n_l的关系替换为l与$l+1$的关系，就可以得到：

$$
\delta_i^{(l)} = \left(\sum_{j=1}^{s_{l+1}} W_{ji}^{(l)} \delta_j^{(l+1)} \right) f'(z_i^{(l)})
$$

以上逐次从后向前求导的过程即为“反向传导”的本意所在。

反向传播

4. 计算我们需要的偏导数，计算方法如下：

$$\frac{\partial}{\partial W_{ij}^{(l)}} J(W,b;x,y) = a_j^{(l)} \delta_i^{(l+1)}$$

$$\frac{\partial}{\partial b_i^{(l)}} J(W,b;x,y) = \delta_i^{(l+1)}.$$

反向传播

求 $\frac{\partial}{\partial W_{ij}^{(l)}} J(W)$?

问题拆解: $\frac{\partial}{\partial W_{ij}^{(l)}} J(W) = \frac{\partial J(W)}{\partial Z_i^{(l+1)}} * \frac{\partial Z_i^{(l+1)}}{\partial W_{ij}^{(l)}}$

神经元求和: $Z_i^{(l+1)} = \sum_i^n W_{ij}^{(l)} * a_j^{(l)}$

输出对权值的偏导数:

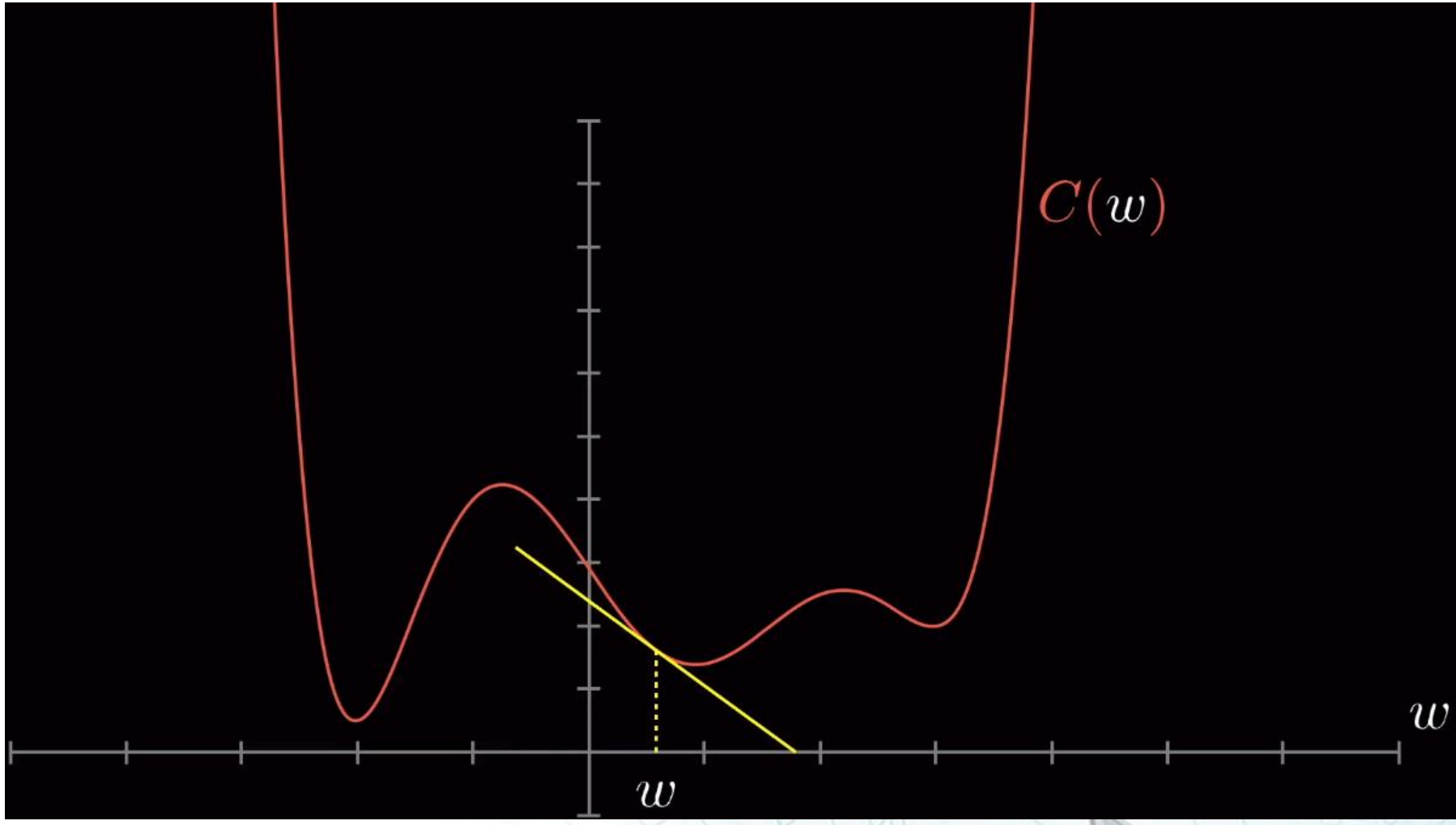
$$\frac{\partial Z_i^{(l+1)}}{\partial W_{ij}^{(l)}} = \frac{\partial \sum_j^n W_{ij}^{(l)} * a_j^{(l)}}{\partial W_{ij}^{(l)}} = a_j^{(l)}$$

设神经元的错误变化率为: $\delta_i^{l+1} = \frac{\partial J(W)}{\partial Z_i^{(l+1)}}$

最终: $\frac{\partial}{\partial W_{ij}^{(l)}} J(W) = \delta_i^{l+1} * a_j^{(l)}$

优化算法

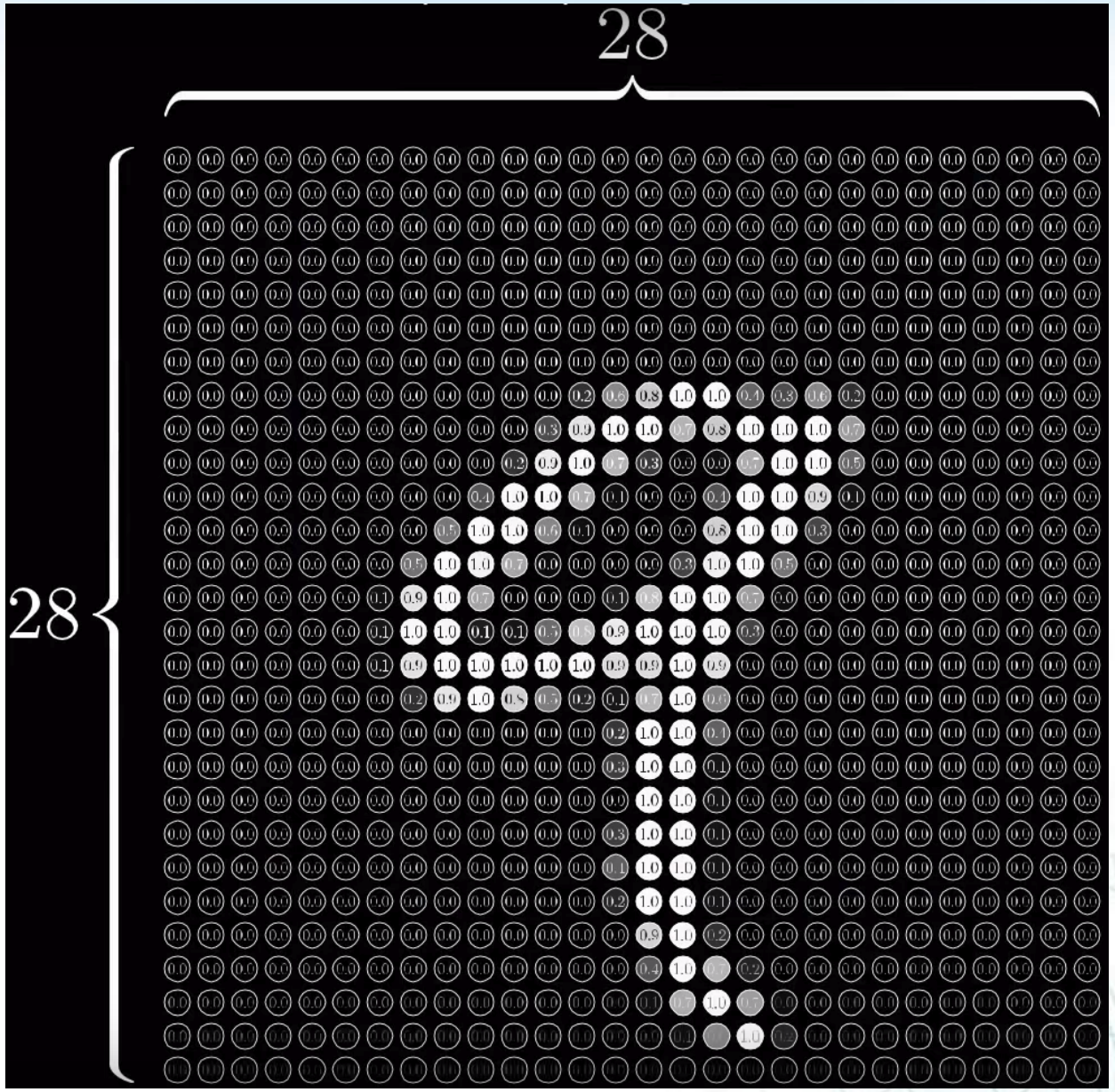
- 梯度下降算法
  - 计算梯度
    - $f'(x)$
  - 变量x向梯度反方向移动
    - $x = x - r * f'(x)$
  - 循环直至满足条件
- Adam，RMSProp算法

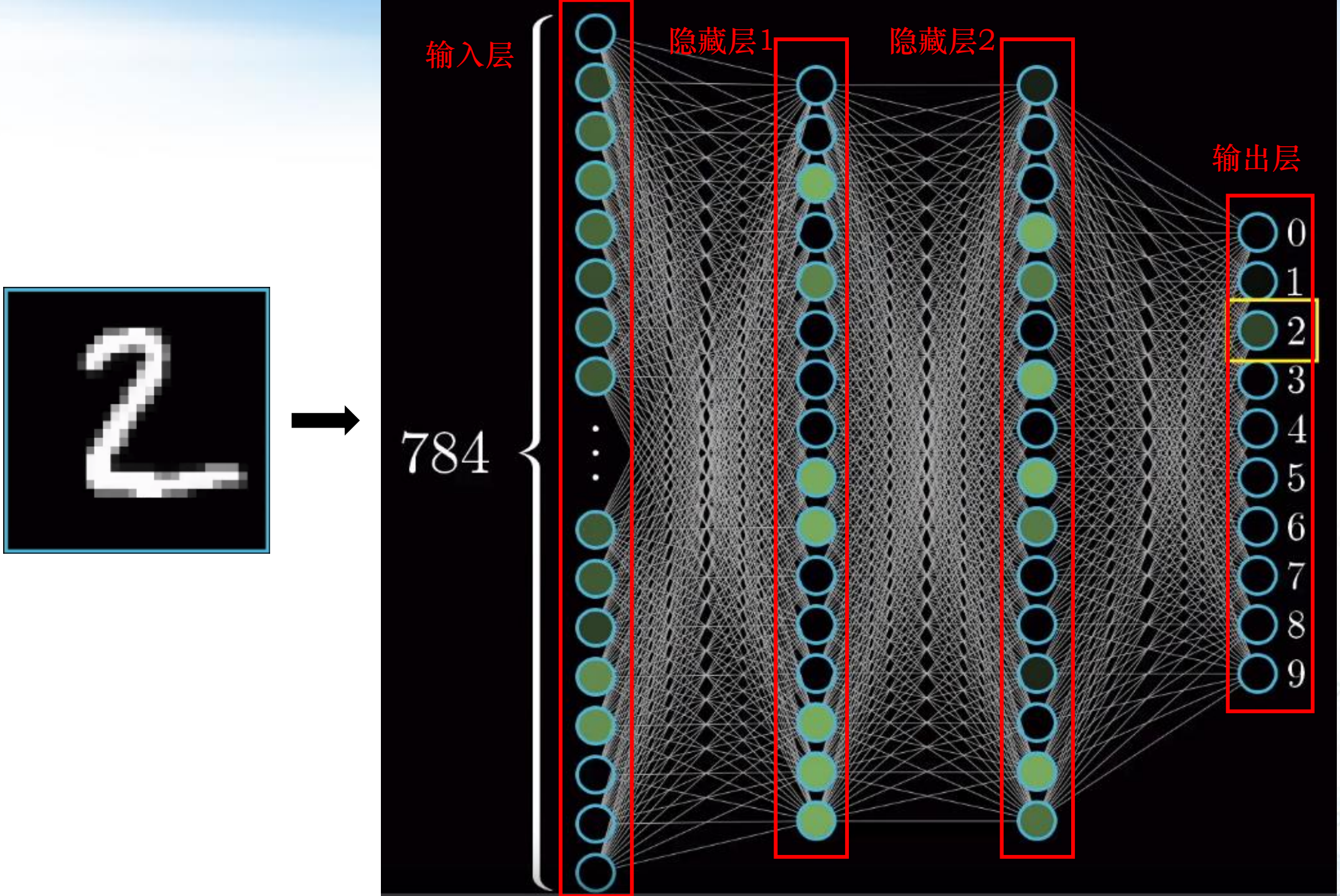


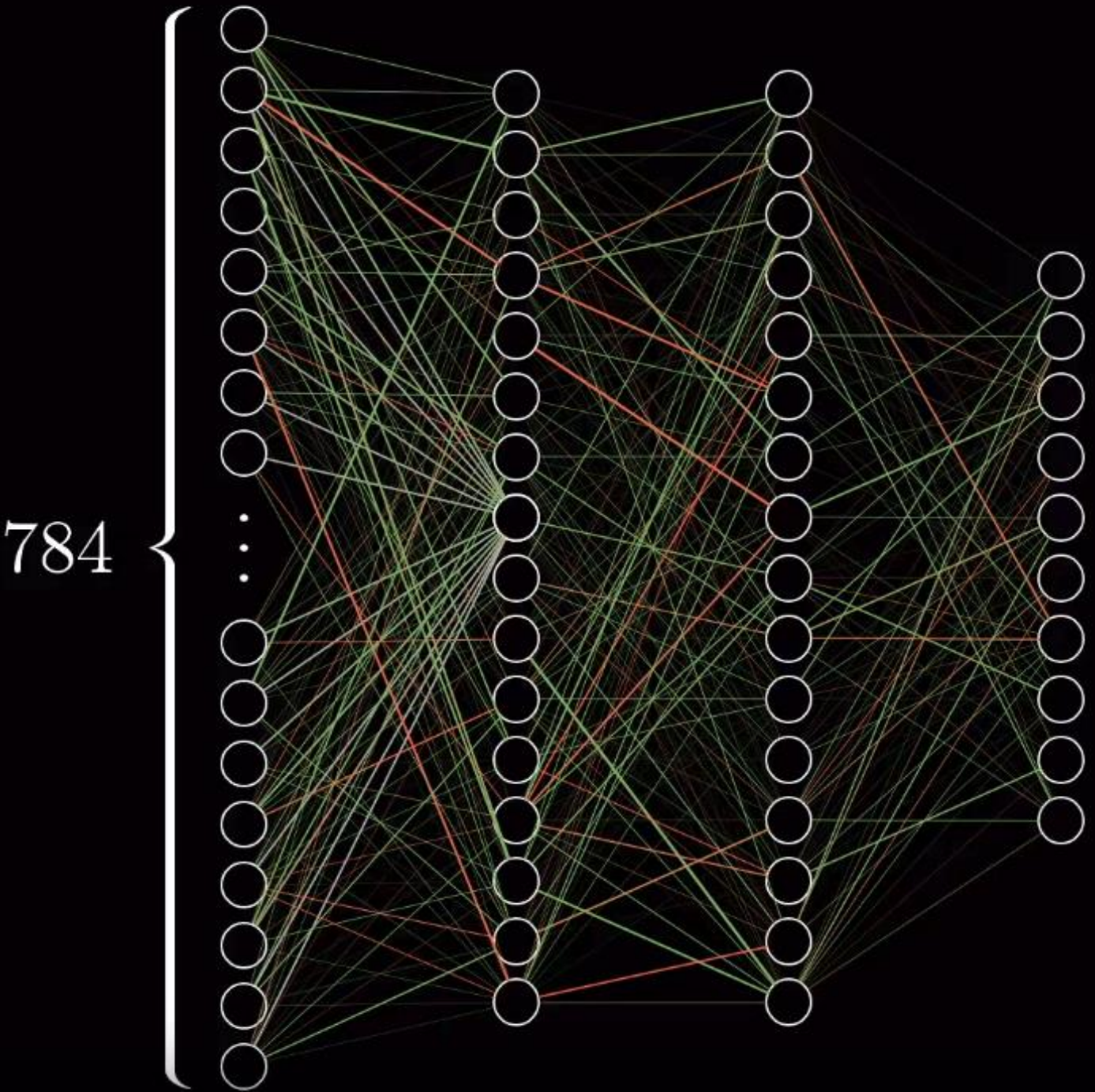
深度学习流行框架：tensorflow

基于数据流图的数值计算开源软件库。数据流图中的点表示数学计算操作，边表示操作与操作之间的高维度数组数据，称为tensor。

深度学习的hello world：MNIST手写数字识别


28
28


输入层
隐藏层1
隐藏层2
输出层
784
0
1
2
3
4
5
6
7
8
9



$784 \times 16 + 16 \times 16 + 16 \times 10$
weights

$16 + 16 + 10$
biases

$13{,}002$

Learning $\rightarrow$ Finding the right weights and biases

Tensorflow:MNIST识别手写数字

```
from tensorflow.examples.tutorials.mnist import input_data

mnist = input_data.read_data_sets("MNIST_data/", one_hot=True)  # 读取MNIST手写数字识别数据集
import tensorflow as tf

X = tf.placeholder(tf.float32, [None, 784], name='x-input')  # tensorflow变量占位符，在执行运算时才传入数据
Y = tf.placeholder(tf.float32, [None, 10], name='y-input')


def init_weights(shape):  # 使用服从正态分布的随机值来初始化参数矩阵
    return tf.Variable(tf.random_normal(shape, stddev=0.01))


w_h = init_weights([784, 16])  # 从输入层到hidden layer1，28x28=784维转化为16维
w_h2 = init_weights([16, 16])  # 从hidden layer1到hidden layer2，16维转化为16维
w_o = init_weights([16, 10])  # 从hidden layer2到输出层，16维转化为10维
```

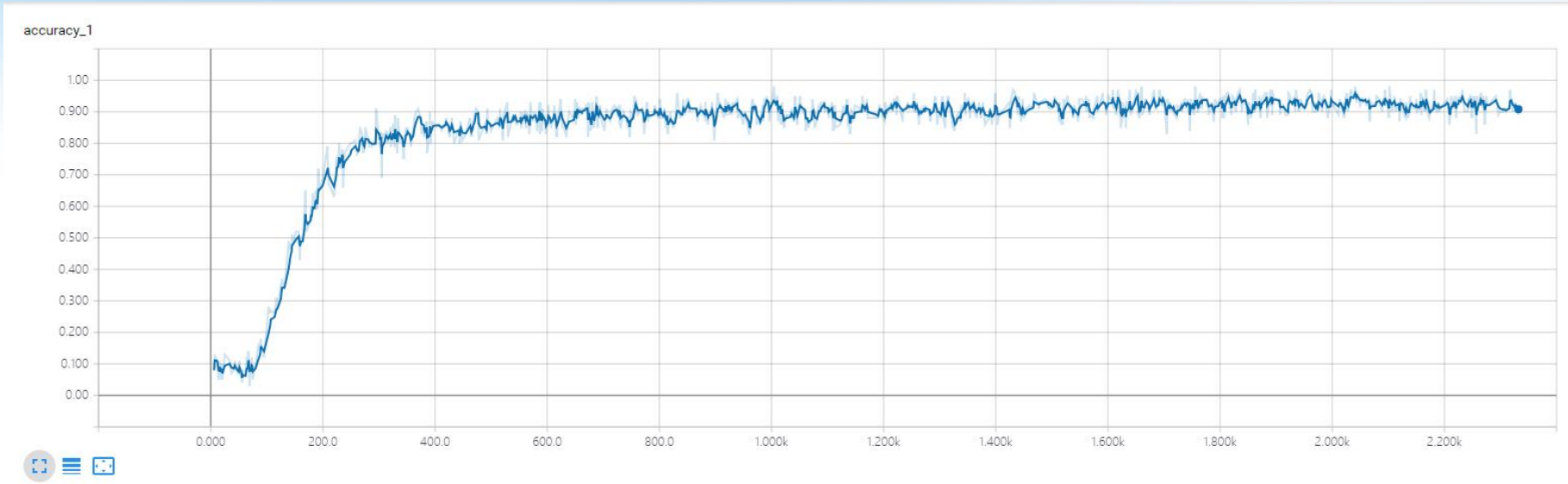
Tensorflow:MNIST识别手写数字

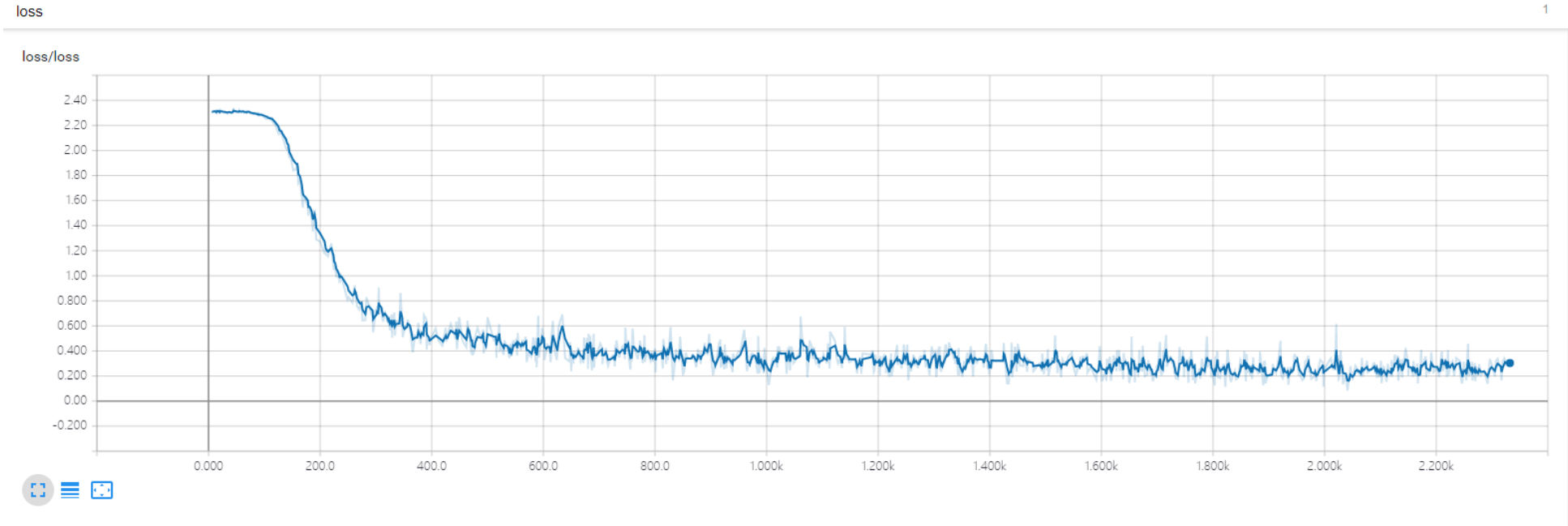
```python
def model(X, w_h, w_h2, w_o):  # 构建神经网络训练模型
    h = tf.nn.relu(tf.matmul(X, w_h))  # hidden layer1是由输入层和权重矩阵1进行矩阵乘法后经过激活函数得到
    h2 = tf.nn.relu(tf.matmul(h, w_h2))  # hidden layer2是由hidden layer1和权重矩阵2进行矩阵乘法后经过激活函数得到
    return tf.matmul(h2, w_o)  # 返回hidden layer2和权重矩阵3的矩阵乘法，即输出层


py_x = model(X, w_h, w_h2, w_o)  # 初始化一个神经网络训练模型
cost = tf.reduce_mean(tf.nn.softmax_cross_entropy_with_logits(logits=py_x, labels=Y))  # 设置改模型的代价函数
train_op = tf.train.RMSPropOptimizer(0.001, 0.9).minimize(cost)  # 设置优化算法，这里是改进后的梯度下降算法
predict_acc = tf.reduce_mean(tf.cast(tf.equal(tf.argmax(py_x, 1), tf.argmax(Y, 1)), tf.float32))  # 计算预测的准确率
epoch_count = 2333  # 需要计算的次数
batch_size = 64  # 每一批的图片数量
```

Tensorflow:MNIST识别手写数字

```
with tf.Session() as sess:
    sess.run(tf.global_variables_initializer())  # 初始化各种变量
    step = 0
    for i in range(epoch_count):
        step += 1
        batch_x, batch_y = mnist.train.next_batch(batch_size)  # 读取相应一批的图片和标签数量
        sess.run(train_op, feed_dict={X: batch_x, Y: batch_y})  # 通过神经网络训练
        if step % 128 == 0:  # 输出训练记录
            loss, acc = sess.run([cost, predict_acc],
                                 feed_dict={X: batch_x, Y: batch_y})
            print("Epoch: {}".format(step), "\tLoss: {:.6f}".format(loss), "\tTraining Accuracy: {:.5f}".format(acc))
    print("Testing Accuracy: {:0.5f}".format(sess.run(predict_acc,
                                                      feed_dict={X: mnist.test.images, Y: mnist.test.labels})))
```

```
Accuracy at step 0: 0.0819
Accuracy at step 128: 0.3761
Accuracy at step 256: 0.7901
Accuracy at step 384: 0.8468
Accuracy at step 512: 0.8702
Accuracy at step 640: 0.886
Accuracy at step 768: 0.8964
Accuracy at step 896: 0.9032
Accuracy at step 1024: 0.9078
Accuracy at step 1152: 0.9036
Accuracy at step 1280: 0.9111
Accuracy at step 1408: 0.9116
Accuracy at step 1536: 0.9141
Accuracy at step 1664: 0.9194
Accuracy at step 1792: 0.9227
Accuracy at step 1920: 0.9265
Accuracy at step 2048: 0.9275
Accuracy at step 2176: 0.9284
Accuracy at step 2304: 0.9313
```





网络结构

- 卷积神经网络（CNN）
  - 全连接层、卷积层、池化层
- 循环神经网络（RNN）
  - 专门处理序列化数据
  - 长短期记忆网络（LSTM）
- CNN+RNN
  - 根据图像语义自动生成摘要

谢谢

Thanks for watching